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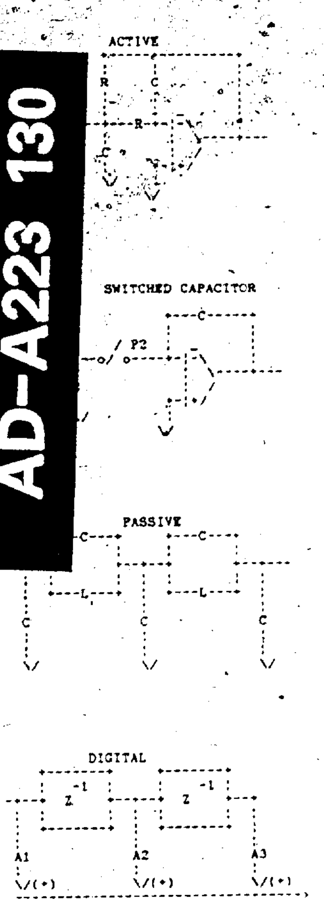

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DESIGN OF SHARP CUTOFF FILTERS WITH LOW QUALITY FACTORS

by

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Electronic filters are used to pass a band of frequencies while rejecting other frequencies. For example, tuning an FM radio is in effect tuning a filter that passes the signal from the desired FM station and blocks the signal from the undesired stations. Advances in technology call for filters with increasingly sharp cutoffs, and these filters are also increasing difficult to manufacture. This difficulty arises from the fact that as the order of the filter is increased, the quality factor Q of each filter section is required to be correspondingly higher. A high Q filter can only be built by using components that closely approximate ideal components, which is often too expensive or not practical. This report presents a method of designing and building filters from low Q components that closely approximate the high Q models.

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Preface

This report details a new method for the design and construction of analog filters so that very restrictive requirements can be met with inexpensive components. The benefits extend to active, digital, and switched capacitor filters in which the high quality factor Q sections would be replaced by multiple low Q sections. The manufacturing process would realize enormous cost savings, in particular in switched capacitor integrated circuit design where a significant reduction in the size of the die could be achieved.

This report was written by Michael G. Ellis, Information Technology Laboratory (ITL), US Army Engineer Waterways Experiment Station (WES), under the supervision of Dr. Windell F. Ingram, Chief, Computer Science Division, and Dr. N. Radhakrishnan, Chief, ITL.

COL Larry B. Fulton, EN, was Commander and Director of WES, and Dr. Robert W. Whalin was Technical Director during the time covered by the research in this paper.

Design of Sharp Cutoff Filters With Low Quality Factors

Introduction

An electronic filter passes certain frequencies whi!e rejecting others. All electronic devices use filters, including radio, TV, radar, microwave, satellite communications, telecommunications, and networks. Filters may be realized using active, passive, or digital components. In recent years, a new type of filter has come into popular use. This filter is called a switched capacitor filter and is basically an active filter modified for precision manufacturing as an integrated circuit.

As a preliminary review of analog and switched capacitor filtors, the following definitions are given. The quality factor, or Q , or a component is defined as the reactance (at the resonant frequency) divided by the resistance. For a capacitor, the Q is

$$Qc = \frac{\text{Parallel Resistance}}{\text{Capacitive Reactance}} \tag{1}$$

Capacitive Q's are almost always ideal since the parallel resistance is very large. For an inductor, the Q is

$$Q_l = \frac{\text{Inductive Reactance}}{\text{Series Resistance}} \tag{2}$$

Inductive Q's are often poor, and designs with inductors should be avoided whenever possible.

Circuit Q's are defined by the equation,

$$\frac{V_o}{V_i} = \frac{W_o^2}{S^2 + \frac{W_o}{Q} S + W_o^2} \tag{3}$$

where

V_o = output voltage

V_i = input voltage

W_o = natural resonant frequency, radians/sec

$S = 2 \cdot \pi \cdot \text{frequency} \cdot \sqrt{-1}$

Circuits with values of $Q > 10$ are increasingly more difficult to build and more unstable with regard to temperature changes, ageing, and manufacturing tolerance. The techniques described in this paper provide a way to design filters with low Q's that approximate the rolloff characteristics of high Q filters, but are easy to manufacture and more tolerant of manufacturing and environmental processes.

Q Compression

Using Q compression, the single high Q section can be replaced by two (or more) lower Q sections, with very little change in the rolloff characteristics. This technique replaces the single section in Equation 3 with two low Q quadratic sections, given by

$$\frac{V_o}{V_i} = \frac{S^2 + AW_oS + BW_o^2}{S^2 + \left(\frac{W_o}{Q_1}\right)S + W_o^2} \qquad \frac{W_o^2}{S^2 + \left(\frac{W_o}{Q_1}\right)S + W_o^2} \tag{4}$$

where Q_1 (the quality factor of section 1) > square root of Q . The coefficients A and B can be determined by equating Equations 3 and 4, using $W_o = 1$. The resulting equation

$$S^4 + \left(A + \frac{1}{Q}\right)S_3 + \left(\frac{A}{Q} + B + 1\right)S^2 + \left(A + \frac{B}{Q}\right)S + B$$

$$= S^4 + \frac{2}{Q_1}S^3 + \left(2 + \frac{1}{Q_1}\right)S^2 + \frac{2}{Q_1}S + 1 \tag{5}$$

with $Q <> Q_1$ cannot be solved exactly; however, the values

$$A = \frac{2}{Q_1} - \frac{1}{Q} \tag{6}$$

and

$$B = 1 \tag{7}$$

provide one approximate solution. This solution increases in accuracy as the value of Q_1 approaches Q . Numerical optimization of W_o , A , and B for desired passband ripple and cutoff characteristics, or optimization of the parameters in other stages of the filter, will further compensate for the effects of the approximation.

Example

Suppose a lowpass filter is required with a passband ripple of 1 db, stopband ripple of 50 db, cutoff frequency of 1 radian/sec, and stopband frequency of 1.05 radians/sec. Since the theoretical order of this filter is 8.07, a 9th order implementation will be used. The Q's of the various stages are

Stage	Q
1	Real pole
2	1.58
3	5.75
4	20.8
5	100.0

Stage 5 can be represented by the normalized quadratic section

$$\frac{V_o}{V_i} = \frac{W_o^2}{S^2 + 0.01\,W_o S + W_o^2}\,, \quad W_o = 1 \tag{8}$$

For some applications, it may be desirable to replace stage 5 with two lower Q stages. If Q_1 is arbitrarily chosen as 20, then the two quadratics can be written as

Stage 5A   Stage 5B

$$\frac{V_o}{V_i} = \frac{S^2 + 0.09\,W_o S + W_o^2}{S^2 + 0.05\,W_o S + W_o^2} \qquad \frac{W_o^2}{S^2 + 0.05\,W_o\,S + W_o^2} \tag{9}$$

where $W_o = 1$ for the normalized filter. A plot of the original filter magnitude response, and the low Q approximation, is shown in Figure 1 with the corresponding group delays in Figure 2. In this example, the -1 db cutoff frequency has shifted to 0.982 radians/sec, with the stopband frequency at 1.028 radians/sec. The cutoff frequency for the entire filter can be scaled to 1 radian/sec and the transition band is still within the 5 percent bandwidth as originally specified for the example. Any value of Q_1 from 10 to 99 could have been used with varying degrees of accuracy versus frequency.

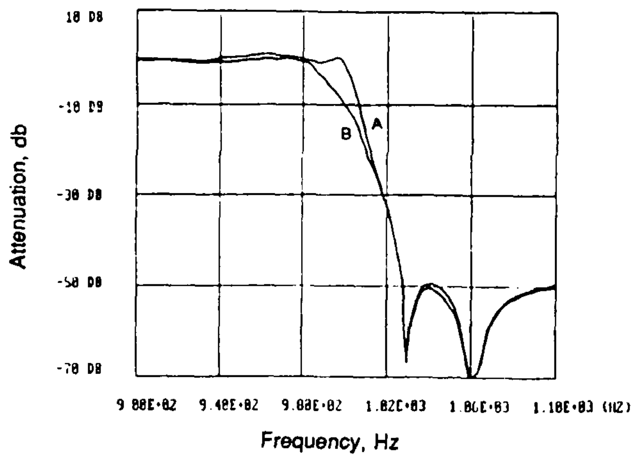

Figure 1. Elliptical filter response (curve A) and low Q approximation (curve B). Significant differences in these two responses occur only over a 12-Hz interval.

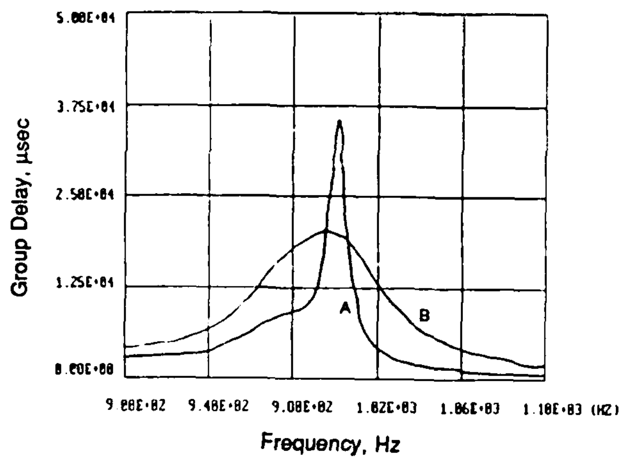

Figure 2. Elliptical filter group delay (curve A) and low Q approximation group delay (curve B).

Error Analysis

An error function, obtained by subtracting Equation 4 from Equation 3, is given by

$$\text{1st degree error} = \frac{\left(\frac{1}{Q} - \frac{1}{Q_1}\right)^2 S W_0^5}{\left(S^2 + S\frac{W_0}{Q} + W_0^2\right)\left(S^2 + S\frac{W_0}{Q_1} + W_0^2\right)^2} \tag{10}$$

The worst-case error for the approximation occurs at $S = W_0$ and is given by

$$\text{Worst-case error} = \left(\frac{1}{Q} = \frac{1}{Q_1}\right)^2 Q Q_1^2 \tag{11}$$

If the error is too large to be tolerable, the error function given in Equation 10 can itself be approximated as part of filter transfer function; however, the resulting complexity in filter implementation usually does not warrant this additional approach. In general, the approximation of the error functions as part of the filter transfer function could continue indefinitely. The general expression for worst-case error is given by

$$\text{General worst-case error} = \left(\frac{1}{Q} - \frac{1}{Q_1}\right)^{2N} Q Q_1^{2N}$$

for N = 1, 2, 3, ... where 2N is the number of low Q quadratic sections to be substituted for the high Q stage in the original filter design.

Summary

Although the example used in this article is a lowpass filter, Q compression is equally valid for bandpass, bandstop, and highpass filters. Q compression can be applied to the final filter, or to the lowpass prototype prior to bandpass transformations.

This technique is completely general with respect to the Q . Filters with Q's of 1,000 or greater can be effectively simulated using much lower Q quadratic sections. Q compression can be applied to active, digital, and switched capacitor filters, but not easily to passive ladder filters because of the complex zeroes that are generated by the substitution. A significant reduction in total capacitance is achievable for switched capacitor integrated circuit design.

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